ADAPTIVE KNOWLEDGE INJECTION FOR MONTE CARLO TREE SEARCH FOR IMPERFECT INFORMATION GAMES

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Supervised by:
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&
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• Why Study Games?
• Magic The Gathering:
  – Large Amount of Uncertainty
  – Large Branching Factor
  – Tactical Play
  – Disambiguation
Magic: The Gathering

• 20 Starting Life
• 60 Card Decks From Set of 10,000
• 5 Different Colours: Red, Blue, Green, White, Black
• Different Types Of Cards:
  – Land
  – Creature
  – Instant
- Produce Mana
- No Cost
- Can Only Play 1 A Turn
Creature

- Used for Combat
- Has Power and Toughness
- Summoning Sickness
Attacking & Blocking

- 1 Combat Phase Per Turn
- Turn Player Chooses attackers
- Opponent which creatures to block
- Unblocked creatures deal damage to opponents
• Creatures deal damage equal to their power to enemy creatures toughness simultaneously.
Attacking & Blocking

- Willow Elf
- Elite Vanguard
Attacking & Blocking

- Another Example...
• Another Example…

Deals 2 damage. Receives 2 damage

Deals 2 damage to creature 1 and 1 damage to creature 2.

Receives 4 damage

Deals 2 damage. Receives 1 damage
• Another Example...
Instant

• Once Off
• Play During any Phase of the game
• Make the game highly interactive
Magic: The Gathering

• Turn Based
• Different Phases
  – Upkeep
  – Draw
  – Main
  – Combat (Attack, Block)
  – Second Main
  – End
Sample Game

- Jeremy has rolled 14
- Bob has rolled 11
- Jeremy Chooses to play first
Sample Game

Opponent life total: 10
My life total: 10
Cards In Hand

- Foundry Street Denizen
- Satyr Hoplite
- Coordinated Assault
- War-Name Aspirant

Foundry Street Denizen
Creature — Goblin Warrior
Whenever another red creature enters the battlefield under your control, Foundry Street Denizen gets +1/+0 until end of turn.

Satyr Hoplite
Creature — Satyr Soldier
Heroic — Whenever you cast a spell that targets Satyr Hoplite, put a +1/+1 counter on Satyr Hoplite.

Coordinated Assault
Instant
Up to two target creatures each get +1/+0 and gain first strike until end of turn.

War-Name Aspirant
Creature — Human Warrior
Raid — War-Name Aspirant enters the battlefield with a +1/+1 counter on it if you attacked with a creature this turn. War-Name Aspirant can’t be blocked by creatures with power 1 or less.

Total Cards: 16

2x Foundry Street Denizen
2x Satyr Hoplite
1x Coordinated Assault
1x War-Name Aspirant
Option 1:
• Turn 1: Foundry Street Denizen

• Turn 2: 2 x Satyr Hoplite, Attack for 3

• Turn 3: Coordinated Assault, Attack for 7
Option 2:
• Turn 1: Satyr Hoplite
• Turn 2: Foundry Street Denizen, Satyr Hoplite, Attack for 1?
Sample Game

Opponent life total: 10

My life total: 10
Sample Game

Opponent life total: 10

My life total: 10
Sample Game

Opponent life total: 10

My life total: 10
Sample Game

Opponent life total: 10

My life total: 10
Sample Game

Opponent life total: 10

My life total: 10
Sample Game

Opponent life total: 10

My life total: 10
Sample Game

Opponent life total: 10
5
2

My life total: 10
5
3
Sample Game

Opponent life total: 10

My life total: 10
Sample Game

Opponent life total: 10

My life total: 10

Tormented Hero

Creature — Human Warrior

Tormented Hero enters the battlefield tapped.

Heroic — Whenever you cast a spell that targets Tormented Hero, each opponent loses 1 life. You gain life equal to the life lost this way.
Sample Game

Opponent life total: 10

My life total: 10
Sample Game

Opponent life total: 10

My life total: 10

5

2

5

3
Sample Game

Opponent life total: 10

My life total: 10
Sample Game

Opponent life total: 10

My life total: 10
Sample Game

Opponent life total: 10

My life total: 10
Sample Game

Opponent life total: 10

My life total: 10
Sample Game

Opponent life total: 10

My life total: 10
Sample Game

Opponent life total: 10

My life total: 10
Sample Game

Opponent life total: 10

My life total: 10
Sample Game

Opponent life total: 10

My life total: 10
Sample Game

Opponent life total: 10

My life total: 10
Sample Game

Opponent life total: 7

My life total: 10
Sample Game

Opponent life total: 7

My life total: 10
Sample Game

Opponent life total: 7

My life total: 10
Sample Game

Opponent life total: 7

My life total: 10
Sample Game

Opponent life total: 7

My life total: 10
Sample Game

Opponent life total: 7

My life total: 10
Block Analysis

Single Block

[Image of game cards]
Block Analysis

Single Block
Block Analysis

Double Block
Block Analysis

Double Block
Sample Game

Opponent life total: 7

My life total: 10
Sample Game

Opponent life total: 7

My life total: 8
Sample Game

Opponent life total: 7

My life total: 8
Sample Game

Opponent life total: 7

My life total: 8
Sample Game

Opponent life total: 7

My life total: 8
Sample Game

Opponent life total: 7

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My life total: 8
Sample Game

Opponent life total: 7

My life total: 8
Sample Game

Opponent life total: 7

My life total: 8
Sample Game

Opponent life total: 7

My life total: 8

Total:

- Opponent: 5
- My: 5

Remaining:

- Opponent: 1
- My: 1
Sample Game

Opponent life total: 7

My life total: 8
Sample Game

Opponent life total: 7

My life total: 8
Sample Game

Opponent life total: 7

My life total: 8
Sample Game

**Opponent life total:** 7

**My life total:** 8

- Opponent: 5
- My life: 5
- Opponent: 1
- My life: 1
Sample Game

Opponent life total: 7

My life total: 8

Bile Blight

Target creature and all other creatures with the same name as that creature get -3/-3 until end of turn.

Not an arrow loosed, javelin thrown, nor sword raised. None were needed.
Sample Game

Opponent life total: 7

My life total: 8
Sample Game

Opponent life total: 7

My life total: 8
Sample Game

Opponent life total: 6

My life total: 8
Sample Game

Opponent life total: 6

My life total: 8
Sample Game

Opponent life total: 6

My life total: 8
Sample Game

Opponent life total: 6

My life total: 8
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My life total: 8
Sample Game

Opponent life
total: 6

My life
total: 8
Sample Game

Opponent life total: 6

My life total: 8
Sample Game

Opponent life total: 6

My life total: 8

Blood-Chin Rager
Creature — Human Warrior

Whenever Blood-Chin Rager attacks, each Warrior creature you control can’t be blocked this turn except by two or more creatures.

Kolaghan blades rarely stay clean for long.
Sample Game

Opponent life total: 6

My life total: 8
Sample Game

Opponent life total: 6

My life total: 8
Sample Game

Opponent life
  total: 6

My life
  total: 8
Sample Game

Opponent life total: 6

My life total: 8
Block Analysis

**Tormented Hero**
- Deals 2 damage.
- Receives 3 damage

**War-Name Aspirant**
- Deals 3 damage.
- Receives 2 damage
Block Analysis

[Image of a card with text]

[Image of another card with text]
Sample Game

Opponent life total: 6

My life total: 8
Sample Game

Opponent life total: 6

My life total: 6
Opponent life total: 6

My life total: 6

Sample Game
Sample Game

Opponent life total: 6

My life total: 6

Lightning Bolt

Lightning Bolt deals 3 damage to target creature or player.
Sample Game

Opponent life
total: 6

My life
total: 6
Opponent life total: 6

My life total: 6
Sample Game

Opponent life total: 6

My life total: 6
Opponent life
total: 6

Option 1:
• Attack With Both our creatures, hope Bob doesn’t block.
• Lightning bolt to kill Bob
Attack Analysis

Opponent life total: 6

Option 1:

- Attack With Both our creatures, hope Bob doesn’t block.
- Lightning bolt to kill Bob

Bob Blocks = Trade 2/1 for 1/1 and 3/2 for 2/2
Opponent life total: 6

Option 2:

- Lightning Bolt Creature 1.
- Attack with War-Name Aspirant
- Leave Bob at 3
Sample Game

Opponent life total: 6

My life total: 6
Sample Game

Opponent life total: 6

My life total: 6
Sample Game

Opponent life total: 6

My life total: 6
Sample Game

Opponent life total: 6

My life total: 6
Sample Game

Opponent life total: 6

My life total: 6

When Mogis’s Marauder enters the battlefield, up to X target creatures each gain intimidate and haste until end of turn, where X is your devotion to black. (Each \( \bullet \) in the mana costs of permanents you control counts toward your devotion to black.)
Sample Game

Opponent life total: 6

My life total: 6

Can't be blocked Can't be blocked Can't be blocked Can't be blocked
Sample Game

Opponent life total: 6

My life total: -2

Can't be blocked
Why did we lose?

- Playing against black, we should have been suspicious of bile blight
Why did we lose?

- People playing these 3 cards are usually playing the fourth
So What?

- Modelling our opponent’s hand is important
- Guessing randomly doesn’t help
- Can guess based on opponent plays
Imperfect Info Games

• What are they?
• General Methods for creating
  – Abstraction
  – Sampling
Abstraction

- Information Abstraction
- Action Abstraction
- Phase Abstraction
Information Abstraction

• Merge Similar States of the Game
• Agents can’t distinguish between merged states
• Eg: Merge hands of similar strength into buckets
• Works with Poker, not so well with Bridge, Skat and MTG
Action Abstraction

- Group Distinct Actions Together
- Works in Texas Hold’em
- Eg: Grouping betting actions
Phase Abstraction

• Some games are played in different phases
• Solve for the phases separately
• MTG seems like a good candidate
Sampling Algorithms

- Traverse Selected Nodes In a Game Tree
- Monte Carlo Tree Search (MCTS)
  - Selection
  - Expansion
  - Simulation
  - Back Propagation
Play for X
MCTS Illustration

Selection

\[
\begin{array}{c|c}
\times & O \\
\hline
\times & X \\
\hline
& O \\
\end{array}
\]
Expansion

```
  1  
 2 3
 4 5
```

5 Unexplored Children
MCTS Illustration

Expansion
Simulation

MCTS Illustration
MCTS Illustration

Simulation

```
  X  |  O
  X  |  X
  X  |  O
```

```
  X  |  O
  X  |  O
```

```
  X  |  O
  X  |  O
```

```
  X  |  O
  X  |  O
```

Simulation

MCTS Illustration
MCTS Illustration

Backpropagation

```
X O
X X
-1/1
```

```
X  O
X  O
```

-1/1
MCTS Illustration

Backpropagation

Iteration 1 Complete
MCTS Illustration

Selection

-1/1
MCTS Illustration

Expansion

-1/1

Unexplored Children
MCTS Illustration

Expansion

-1/1
MCTS Illustration

Simulation

-1/1

Diagram of tic-tac-toe games with outcomes and rewards.
MCTS Illustration

Simulation

-1/1
MCTS Illustration

Simulation

-1/1
Backpropagation
MCTS Illustration

Backpropagation

Iteration 2 Complete
MCTS Illustration

etc…

Iteration 5 complete
MCTS Illustration

Selection

UCB: $\bar{x}_j + \sqrt{\frac{2\ln(n)}{n_j}}$
MCTS Illustration

Selection

\[-1/1\]

\[1/1\]

\[0/1\]

\[1/1\]

\[-1/1\]

\[-1 + \sqrt{\frac{2 \ln(5)}{1}} = 0.7941\]
MCTS Illustration

Selection

-1/1
0.79
41

1/
2.79
41

0/1
1.79
41

1/1
2.79
41

-1/1
0.79
41
MCTS Illustration

Selection

Tie! Break Tie Randomly
MCTS Illustration

Simulation

-1/1  1/1  0/1  1/1  -1/1
MCTS Illustration

Backpropagation

-1/1

1/1

0/1

1/1

-1/1

0/1
MCTS Illustration

Backpropagation

Diagram of tic-tac-toe game states with evaluation values.
MCTS Illustration

Backpropagation

Iteration 6 complete
Continue until some predefined limit...
MCTS For MTG

- Peter Cowling
- Perfect Information Mote Carlo Search (PIMC)
  - Sample Possible Worlds
  - Find Beset Move From Worlds
  - Play most chosen move
• Guess opponent’s hand
• Guess exactly how our opponent will draw.
• Guess exactly how we will draw
• Collection of guesses makes a “World”
• Run MCTS on the “World”
• Repeat above
• Used PIMC
• MCTS used Strong Heuristic Player + Random
• Downsides:
  – Need to update rules for the agent when card base changes
  – Provided opponent with exact composition of opponent’s deck (Unrealistic)
Our Approach

• Use a sampling algorithm
• Use Disambiguation to gain knowledge
• Use the knowledge to predict opponent’s hand
• Random Predictions will lead to bad estimates/bad play
• Better estimate = Reduced # states => Better sampling
Card Correlation

• What does this mean?
• Required Reasonable Decks
  – Required Reasonable Agents
  – Deck Building Procedure
• Play off against pre made deck
• Take top 1000 decks, create card correlation matrix
# Card Correlation

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<td>-0.9709</td>
<td>-1.2372</td>
<td>-1.6507</td>
<td>1.1294</td>
<td>-0.8534</td>
<td>-1.2779</td>
<td>-1.5658</td>
<td>-1.3457</td>
<td>-0.9126</td>
<td>-1.3537</td>
<td>1.495</td>
<td>4.8489</td>
<td>0.1903</td>
</tr>
</tbody>
</table>

**L**: Land  
**C1**: Creature 1  
**C2**: Creature 2  
**I**: Instant  
**G**: Green  
**U**: Blue  
**R**: Red  
**W**: White  
**B**: Black
• **Should do**
  – Better than no knowledge
  – Worse than perfect knowledge
Prior Knowledge Agent

Distribution After seeing a red land

Distribution After seeing a red land, red creature
Prior Knowledge Agent

Distribution After seeing a few more red cards...

Eventually...
Current Results

![Graph showing win rate over the number of simulations for different knowledge conditions.](image)
Future Work

- Carry Knowledge Across Games
- Run Experiments With Different Decks
- Model Distributions Over Decks Instead of cards